Revisiting the tunneling spectrum and information recovery of a general charged and rotating black hole

Ge-Rui Chen* and Yong-Chang Huang

Institute of Theoretical Physics, Beijing University of Technology, Beijing, 100124, China

Abstract

In this paper we revisit the tunneling spectrum of a charged and rotating black hole–Kerr-Newman black hole by using Parikh and Wilczek' s tunneling method and get the most general result compared with the works [9, 10]. We find an ambiguity in Parikh and Wilczek' s tunneling method, and give a reasonable description. We use this general spectrum to discuss the information recovery based on the Refs. [11–13]. For the tunneling spectrum we obtained, there exit correlations between sequential Hawking radiations, information can be carried out by such correlations, and the entropy is conserved during the radiation process. So we resolve the information loss paradox based on the methods [11–13] in the most general case.

PACS: 04.70.Dy; 04.70.-s; 03.65.Sq; 03.67.-a

Keywords: Kerr-Newman black hole, tunneling spectrum, correlation, mutual information, entropy conservation process

*Electronic address: chengerui@emails.bjut.edu.cn

I. INTRODUCTION

Since Hawking radiation [1, 2] was discovered, black hole physics has been greatly developed. People know that black hole is not the object from which gravity prevents anything, including light, from escaping, and it can radiate thermal spectrum. Black holes are not the final state of stars, and, with the emission of Hawking radiation, they lose energy, shrink, and eventually evaporate completely. However, because the radiation is a purely thermal spectrum, it also sets up a disturbing and difficult information loss problem: what happens to information during black hole evaporation? This scenario is inconsistent with the unitary principle of quantum mechanics [3–6].

The semi-classical method which models Hawking radiation as a tunneling process was proposed by Parikh and Wilczek [7, 8] in 2001. This method uses WKB approximation, and calculates the imaginary part of the action $ImI = \int_{r_{in}}^{r_{out}} p_r dr$ for the classically forbidden process across the horizon. The tunneling probability is given by $\Gamma \propto \exp(-2ImI)$. They found that considering the back reaction of the emission particle to the spacetime, a non-thermal spectrum is given, which supports the underlying unitary theory. After that, there were many works to generalize this method to other different kinds of black holes and all got the same results. The most general case is the charged particles' tunneling from a charged and rotating black hole–Kerr-Newman black hole [9, 10].

In 2009, Refs. [11–13] gave more detailed discussions about Parikh and Wilczek's non-thermal spectrum. Their results are that there exist correlations among sequential Hawking radiations, the correlation is equal to mutual information, and black hole radiation is an entropy conservation process. In Ref. [12], they discussed the properties of charged particles' tunneling spectrum from Kerr-Newman black hole by using the tunneling spectrum in Refs. [9, 10]. We find that this spectrum is not the most general case, and is just the special case for $j = a\omega$ (see the end of section 2). We revisit the calculation and get the most general tunneling spectrum (13). We also find an ambiguity in Parikh and Wilczek's method, and give a reasonable description. Then we use the tunneling spectrum obtained to discuss information conservation and generalize Refs. [11–13]' results to the most general case.

This paper is organized as follows. In Section 2, we revisit the tunneling method to get a general non-thermal spectrum from Kerr-Newman black hole. In Section 3, we investigate the properties of this non-thermal spectrum. In the last Section, we give some discussions

and conclusions.

II. REVISITING PARIKH AND WILCZEK' S TUNNELING METHOD FROM KERR-NEWMAN BLACK HOLE

We recalculate the tunneling spectrum from a general charged and rotating black hole–Kerr-Newman black hole. Although there are some works [9, 10] about this problem, our discussion has new insights and gets the most general result. We will use directly some results already in Ref. [10].

According to WKB approximation, the emission rate Γ can be given as

$$\Gamma \sim \exp(-2ImI), \tag{1}$$

where I is the action of the emitting particle. In the Painlevé-Kerr-Newnan coordinate system, the action for the classically forbidden trajectory of an emitting particle is

$$\begin{aligned} I &= \int [p_r dr - p_{A_t} dA_t - p_\phi d\phi] \\ &= \int [p_r dr - \frac{p_{A_t}\dot{A}_t}{\dot{r}} dr - \frac{p_\phi \dot{\phi}}{\dot{r}} dr] \\ &= \int\int [dp_r - \frac{\dot{A}_t dp_{A_t}}{\dot{r}} - \frac{\dot{\phi} dp_\phi}{\dot{r}}] dr. \end{aligned} \tag{2}$$

For the emitting particle, the Hamilton's equations are

$$\begin{aligned} \dot{r} &= \frac{dH}{dp_r}|_{(r;A_t,p_{A_t};\phi,p_\phi)} = \frac{d\omega}{dp_r}, \\ \dot{A}_t &= \frac{dH}{dp_{A_t}}|_{(A_t;r,p_r;\phi,p_\phi)} = \frac{V_0 dq}{dp_{A_t}}, \end{aligned} \tag{3}$$

so we have

$$dp_r = \frac{d\omega}{\dot{r}}, \ \ \dot{A}_t dp_{A_t} = V_0 dq, \tag{4}$$

where ω, q are the energy and charge of the emitting particle. V_0 is the electrostatic potential on the outer event horizon $r = r_+$,

$$V_0 = \frac{Q r_+}{r_+^2 + a^2}, \tag{5}$$

where Q and $a = \frac{J}{M}$ are the charge and angular momentum of unit mass of black hole respectively. So we can get

$$\begin{aligned} ImI &= Im\int\int[\frac{d\omega}{\dot{r}} - \Omega\frac{dj}{\dot{r}} - \frac{V_0 dq}{\dot{r}}]dr \\ &= \int\int\frac{dr}{\dot{r}}[d\omega - \Omega dj - V_0 dq], \end{aligned} \tag{6}$$

where $\Omega = \frac{a}{r_+^2 + a^2}$ is the angular velocity of the horizon and $p_\varphi = j$ is the angular momentum of the emitting particle. We consider the s-wave, so $\dot{\phi} = \Omega$.

In Ref [10]. the expression of $\dot{r}$ for the charged particle is

$$\dot{r} = \frac{\Delta}{2}\sqrt{\frac{\rho^2}{(\rho^2 - \Delta)[(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta]}}, \tag{7}$$

where

$$\begin{aligned} \Delta &= r^2 + a^2 + Q^2 - 2Mr = (r - r_+)(r - r_-), \\ r_\pm &= M \pm \sqrt{M^2 - a^2 - Q^2}, \\ \rho^2 &= r^2 + a^2\cos^2\theta. \end{aligned} \tag{8}$$

Putting Eq. (7) into Eq. (6), we have

$$\begin{aligned} ImI &= Im\int\int\frac{2\sqrt{(\rho^2 - \Delta)[(r^2 + a^2)^2 - \Delta a^2\sin^2\theta]}}{(r - r_+)(r - r_-)\sqrt{\rho^2}}dr(d\omega - \Omega dj - V_0 dq) \\ &= Im\int \pi i 2\frac{r_+^2 + a^2}{r_+ - r_-}(d\omega - \Omega dj - V_0 dq) \\ &= 2\pi\int\frac{r_+^2 + a^2}{r_+ - r_-}(d\omega - \Omega dj - V_0 dq) \\ &= 2\pi\int\frac{r_+^2 + a^2}{r_+ - r_-}d\omega - \frac{a}{r_+ - r_-}dj - \frac{Qr_+}{r_+ - r_-}dq, \end{aligned} \tag{9}$$

where the integral of r is done by deforming the contour around the pole in the second equality.

When particle's self-gravitation is taken into account we should replace M, J, Q with $M - \omega, J - j, Q - q$ in Eq. (9),

$$\begin{aligned} ImI &= \pi\int_{(0,0,0)}^{(\omega,j,q)}[2(M - \omega) + \frac{2(M - \omega)^3 - (Q - q)^2(M - \omega)}{\sqrt{(M - \omega)^4 - (J - j)^2 - (Q - q)^2(M - \omega)^2}}]d\omega \\ &\quad -\frac{J - j}{\sqrt{(M - \omega)^4 - (J - j)^2 - (Q - q)^2(M - \omega)^2}}dj \\ &\quad -[(Q - q) + \frac{(Q - q)(M - \omega)^2}{\sqrt{(M - \omega)^4 - (J - j)^2 - (Q - q)^2(M - \omega)^2}}]dq. \end{aligned} \tag{10}$$

It is easy to find that

$$\begin{aligned} ImI &= -\frac{1}{2}\int[\frac{\partial(\Delta S)}{\partial\omega}d\omega + \frac{\partial(\Delta S)}{\partial j}dj + \frac{\partial(\Delta S)}{\partial q}dq] \\ &= -\frac{1}{2}\Delta S, \end{aligned} \tag{11}$$

where

$$\begin{aligned} \Delta S &= \pi[2(M-\omega)^2 - (Q-q)^2 + 2\sqrt{(M-\omega)^4 - (J-j)^2 - (Q-q)^2(M-\omega)^2}] \\ &\quad -\pi[2M^2 - Q^2 + 2\sqrt{M^4 - J^2 - Q^2M^2}] \end{aligned} \tag{12}$$

is the change of black hole entropy after emitting a particle with (ω, j, q). So the tunneling rate is

$$\begin{aligned} \Gamma &\sim \exp(-2ImI) = \exp(\Delta S) \\ &= \exp(\pi[2(M-\omega)^2 - (Q-q)^2 + 2\sqrt{(M-\omega)^4 - (J-j)^2 - (Q-q)^2(M-\omega)^2}] \\ &\quad -\pi[2M^2 - Q^2 + 2\sqrt{M^4 - J^2 - Q^2M^2}]). \end{aligned} \tag{13}$$

This is the general result for the tunneling spectrum. Refs. [9, 10] only give a special case with $j = a\omega$,

$$\begin{aligned} \Gamma &\sim \exp(-2ImI) \\ &= \exp(\pi[2(M-\omega)^2 - (Q-q)^2 + 2(M-\omega)\sqrt{(M-\omega)^2 - a^2 - (Q-q)^2}] \\ &\quad -\pi[2M^2 - Q^2 + 2M\sqrt{M^2 - a^2 - Q^2}]). \end{aligned} \tag{14}$$

Let us give some discussions. In the original paper of Parikh and Wilczek's tunneling method [7, 8] and many other followed works [9, 10], for the Hamilton's equation $\dot{r} = \frac{dH}{dp_r}$, they argued that $H = M - \omega$ represents the Hamiltonian of the black hole, and $\dot{r}$ is the radial path of the emitting particles. We think there exists an ambiguity since we could not tell the Hamilton's equation is for what physical object. In Ref. [7–10], a minus sign appears in equation $dH = -d\omega$, and integration around the pole gives the result $-\pi i Res$, where the minus sign comes from the assumption $r_{in} > r_{out}$. This integration is inconsistent with that of another tunneling method–complex path integration method (also called Hamilton-Jacobi method) which gives the integration $\pi i Res$ [14–16]. In our article, we consider that the Hamilton's equation is for the emitting particles, and $H = \omega$ represents the energy of emitting particles. The integrating around pole is $\pi i Res$ which is the same as that of

the complex path integration method. In our opinion the two different tunneling methods should have the same integration around the pole since this integration reflects the barrier in the tunneling process.

III. INFORMATION RECOVERY OF TUNNELING SPECTRUM FROM KERR-NEWMAN BLACK HOLE

In this section we investigate the properties of the tunneling spectrum (13) based on Refs. [11–13]' methods. From Eq. (13), the probability for the emission of a particle with energy, angular momentum and charge (ω_1, j_1, q_1) is

$$\begin{aligned}\Gamma(\omega_1, j_1, q_1) &= \exp(\pi[2(M-\omega_1)^2 - (Q-q_1)^2 + 2\sqrt{(M-\omega_1)^4 - (J-j_1)^2 - (Q-q_1)^2(M-\omega_1)^2}] \\ &\quad -\pi[2M^2 - Q^2 + 2\sqrt{M^4 - J^2 - Q^2M^2}]). \end{aligned} \tag{15}$$

And the probability for the emission of a particle with energy, angular momentum and charge (ω_2, j_2, q_2) is

$$\begin{aligned}\Gamma(\omega_2, j_2, q_2) &= \exp(\pi[2(M-\omega_2)^2 - (Q-q_2)^2 + 2\sqrt{(M-\omega_2)^4 - (J-j_2)^2 - (Q-q_2)^2(M-\omega_2)^2}] \\ &\quad -\pi[2M^2 - Q^2 + 2\sqrt{M^4 - J^2 - Q^2M^2}]). \end{aligned} \tag{16}$$

Note that (ω_1, j_1, q_1) and (ω_2, j_2, q_2) represent two independent emitting particles, so the expressions should have the same form.

Let us consider a process as follows. One particle with energy, angular momentum and charge (ω_1, j_1, q_1) emits, then another particle with energy, angular momentum and charge (ω_2, j_2, q_2) emits. The probability for the emission of the second particle is

$$\begin{aligned}\Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1) &= \exp(\pi[2(M-\omega_1-\omega_2)^2 - (Q-q_1-q_2)^2 \\ &\quad + 2\sqrt{(M-\omega_1-\omega_2)^4 - (J-j_1-j_2)^2 - (Q-q_1-q_2)^2(M-\omega_1-\omega_2)^2}] \\ &\quad - \pi[2(M-\omega_1)^2 - (Q-q_1)^2 \\ &\quad + 2\sqrt{(M-\omega_1)^4 - (J-j_1)^2 - (Q-q_1)^2(M-\omega_1)^2}]), \end{aligned} \tag{17}$$

which is the conditional probability and is different from the independent probability (16).

So the emitting probability for the two successive emissions can be calculated as

$$
\begin{aligned}
\Gamma(\omega_1, j_1, q_1, \omega_2, j_2, q_2) &\equiv \Gamma(\omega_1, j_1, q_1)\Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1) \\
&= \exp(\pi[2(M-\omega_1)^2 - (Q-q_1)^2 \\
&\quad +2\sqrt{(M-\omega_1)^4 - (J-j_1)^2 - (Q-q_1)^2(M-\omega_1)^2}] \\
&\quad -\pi[2M^2 - Q^2 + 2\sqrt{M^4 - J^2 - Q^2M^2}]) \\
&\quad \times \exp(\pi[2(M-\omega_1-\omega_2)^2 - (Q-q_1-q_2)^2 \\
&\quad +2\sqrt{(M-\omega_1-\omega_2)^4 - (J-j_1-j_2)^2 - (Q-q_1-q_2)^2(M-\omega_1-\omega_2)^2}] \\
&\quad -\pi[2(M-\omega_1)^2 - (Q-q_1)^2 \\
&\quad +2\sqrt{(M-\omega_1)^4 - (J-j_1)^2 - (Q-q_1)^2(M-\omega_1)^2}]) \\
&= \exp(\pi[2(M-\omega_1-\omega_2)^2 - (Q-q_1-q_2)^2 \\
&\quad +2\sqrt{(M-\omega_1-\omega_2)^4 - (J-j_1-j_2)^2 - (Q-q_1-q_2)^2(M-\omega_1-\omega_2)^2}] \\
&\quad -\pi[2M^2 - Q^2 + 2\sqrt{M^4 - J^2 - Q^2M^2}]).
\end{aligned} \tag{18}
$$

The last equality is $\Gamma(\omega_1+\omega_2, j_1+j_2, q_1+q_2)$, so we have

$$
\Gamma(\omega_1, j_1, q_1, \omega_2, j_2, q_2) = \Gamma(\omega_1+\omega_2, j_1+j_2, q_1+q_2). \tag{19}
$$

This is an important relationship which tells us that the probability for two particles emitting successively with energy, angular momenta and charges (ω_1, j_1, q_1) and (ω_2, j_2, q_2) is the same as the probability for one particle emitting with energy, angular momentum and charge $(\omega_1+\omega_2, j_1+j_2, q_1+q_2)$. It is easy to see that

$$
\begin{aligned}
\Gamma(\omega_1, j_1, q_1, \omega_2, j_2, q_2, \cdots, \omega_i, j_i, q_i) &= \Gamma(\omega_1, j_1, q_1)\Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1) \\
&\quad \times \cdots \times \Gamma(\omega_i, j_i, q_i|\omega_1, j_1, q_1, \cdots, \omega_{i-1}, j_{i-1}, q_{i-1}) \\
&= \Gamma(\omega_1+\cdots+\omega_i, j_1+\cdots+j_i, q_1+\cdots+q_i).
\end{aligned} \tag{20}
$$

This is an important relationship we will use later.

For the Hawking radiation, the correlation between the two sequential emissions can be calculated as [11–13]

$$
\begin{aligned}
\ln\Gamma(\omega_1+\omega_2, j_1+j_2, q_1+q_2) - \ln[\Gamma(\omega_1, j_1, q_1)\Gamma(\omega_2, j_2, q_2)] &= \ln\frac{\Gamma(\omega_1+\omega_2, j_1+j_2, q_1+q_2)}{\Gamma(\omega_1, j_1, q_1)\Gamma(\omega_2, j_2, q_2)} \\
&= \ln\frac{\Gamma(\omega_1, j_1, q_1)\Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1)}{\Gamma(\omega_1, j_1, q_1)\Gamma(\omega_2, j_2, q_2)} \\
&= \ln\frac{\Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1)}{\Gamma(\omega_2, j_2, q_2)} \neq 0,
\end{aligned} \tag{21}
$$

which shows that the two emissions are statistically dependent, that is to say, there are correlations between sequential Hawking radiations.

Like Eq. (17) the conditional probability $\Gamma(\omega_i, j_i, q_i|\omega_1, j_1, q_1, \cdots, \omega_{i-1}, j_{i-1}, q_{i-1})$ is the tunneling probability of a particle emitting with energy, angular momentum and charge (ω_i, j_i, q_i) after a sequence of radiation from $1 \to (i-1)$, so conditional entropy taken away by this tunneling particle is given by

$$S(\omega_i, j_i, q_i|\omega_1, j_1, q_1, \cdots, \omega_{i-1}, j_{i-1}, q_{i-1}) = -\ln \Gamma(\omega_i, j_i, q_i|\omega_1, j_1, q_1, \cdots, \omega_{i-1}, j_{i-1}, q_{i-1}). \tag{22}$$

The mutual information for the emissions of two particles with energy, angular momenta and charges (ω_1, j_1, q_1) and (ω_2, j_2, q_2) is defined as [11–13]

$$\begin{aligned} S(\omega_2, j_2, q_2 : \omega_1, j_1, q_1) &\equiv S(\omega_2, j_2, q_2) - S(\omega_2, j_2, q_2|\omega_1, j_1, q_1) \\ &= -\ln \Gamma(\omega_2, j_2, q_2) + \ln \Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1) \\ &= \ln \frac{\Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1)}{\Gamma(\omega_2, j_2, q_2)}, \end{aligned} \tag{23}$$

which shows that mutual information is equal to the correlation between the sequential emissions, that is to say, the information is encoded in the correlations between Hawking radiations.

Let us calculate the entropy carried out by Hawking radiations. The entropy carried out by the first emitting particle with energy, angular momentum and charge (ω_1, j_1, q_1) is

$$S(\omega_1, j_1, q_1) = -\ln \Gamma(\omega_1, j_1, q_1). \tag{24}$$

The conditional entropy carried out by the second emitting particle after the first emission is

$$S(\omega_2, j_2, q_2|\omega_1, j_1, q_1) = -\ln \Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1). \tag{25}$$

So the total entropy carried out by the two sequential emissions is

$$S(\omega_1, j_1, q_1, \omega_2, j_2, q_2) = S(\omega_1, j_1, q_1) + S(\omega_2, j_2, q_2|\omega_1, j_1, q_1). \tag{26}$$

Assuming the black hole exhausts after radiating n particles, we have the following relationship

$$\sum_i^n \omega_i = M, \sum_i^n j_i = J, \sum_i^n q_i = Q, \tag{27}$$

where M, J, Q are the mass, angular momentum and charge of the black hole. The entropy carried out by all the emitting particles is

$$\begin{aligned}
S(\omega_1, j_1, q_1, \cdots, \omega_n, j_n, q_n) &= \sum_{i=1}^{n} S(\omega_i, j_i, q_i|\omega_1, j_1, q_1, \cdots, \omega_{i-1}, j_{i-1}, q_{i-1}) \\
&= S(\omega_1, j_1, q_1) + S(\omega_2, j_2, q_2|\omega_1, j_1, q_1) \\
&\quad + \cdots + S(\omega_n, j_n, q_n|\omega_1, j_1, q_1, \cdots, \omega_{n-1}, j_{n-1}, q_{n-1}) \\
&= -\ln\Gamma(\omega_1, j_1, q_1) - \ln\Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1) - \cdots \\
&\quad - \ln\Gamma(\omega_n, j_n, q_n|\omega_1, j_1, q_1, \cdots, \omega_{n-1}, j_{n-1}, q_{n-1}) \\
&= -\ln[\Gamma(\omega_1, j_1, n_1) \times \Gamma(\omega_2, j_2, q_2|\omega_1, j_1, q_1) \times \cdots \times \\
&\quad \Gamma(\omega_n, j_n, q_n|\omega_1, j_1, q_1, \cdots, \omega_{n-1}, j_{n-1}, q_{n-1})] \\
&= -\ln\Gamma(\omega_1 + \omega_2 + \cdots + \omega_n, j_1 + j_2 + \cdots + j_n, q_1 + q_2 + \cdots + q_n) \\
&= -\ln\Gamma(M, J, Q) = \pi[2M^2 - Q^2 + 2\sqrt{M^2 - J^2 - Q^2M^2}] \\
&= S_{BH}, \qquad (28)
\end{aligned}$$

where we use the Eq. (20) in the fifth equation and Eq. (12) in the last equation. The result shows that the entropy carried out by all the emitting particles equals to the black hole original entropy, so the total entropy is conserved in the process of radiation.

IV. DISCUSSIONS AND CONCLUSIONS

In this paper we recalculate the tunneling spectrum of a general charged and rotating black hole–Kerr-Newman black hole by using Parikh and Wilczek' s method and get the most general result (13). A technical difference between our work and former works [9, 10] is that we only use the result $\dot{\phi} = \Omega$ since for the s-wave the particles radiate along the normal direction of cross section, and the condition $j = a\omega$ used in Refs. [9, 10] is unnecessary. We also discuss the ambiguity in Parikh and Wilczek' s tunneling method, and compare it with another tunneling method–complex path integration method (also called Hamilton-Jacobi method). We use this most general result to investigate the back hole information recovery based on Refs. [11–13], and find that for this tunneling spectrum there exit correlations between sequential Hawking radiations, information can be carried out by such correlations, and the entropy is conserved during the radiation process. Therefore, the most general case about back hole information recovery based on the methods [11–13] is given in this paper.

Acknowledgments

This work is supported by National Natural Science Foundation of China (No.11275017 and No.11173028).

---

[1] S. Hawking, Nature **248**(1974)30.

[2] S. Hawking, Commun. Math. Phys. **43**(1975)199.

[3] S. W. Hawking, Phys. Rev. D **14**(1976)2460.

[4] S. W. Hawking, Phys. Rev. D **72**(2005)084013.

[5] C. G. Callan and J. M. Maldacena, Nucl. Phys. B **472**(1996)591.

[6] Samir D. Mathur, Class. Quant. Grav. **26**(2009)224001.

[7] M. K. Parikh, F.Wilczek, Phys. Rev. Lett. **85**(2000)5042.

[8] M. K. Parikh, Int. J. Mod. Phys. D **13**(2004)2351.

[9] Qing-Quan Jiang, Shuang-Qing Wu, Xu Cai, Phys. Rev. D **73**(2006)064003.

[10] J. Zhang and Z. Zhao, Phys. Lett. B **638**(2006)110.

[11] Baocheng Zhang, Qing-yu Cai, Li You, Ming-sheng Zhan, Physics Letters B, **675**(2009)98.

[12] Baocheng Zhang, Qing-yu Cai, Ming-sheng Zhan, Li You, Annals of Physics **326**(2011)350.

[13] Baocheng Zhang, Qing-yu Cai, Ming-sheng Zhan, Li You, Int. J. Mod. Phys.D **22**(2013)1341014. First Award in the 2013 Awards for Essays on Gravitation.

[14] K. Srinivasan and T. Padmanabhan, Phys. Rev. D **60**(1999)24007.

[15] R. Kerner and R. B. Mann, Class. Quant. Grav. **25**(2008)095014.

[16] R. Kerner and R. B. Mann, Phys. Lett. B **665**(2008)277-283.